A Three-Phase Harmonic Power Flow Algorithm Based on A Hybrid Approach

Kuo Lung Lian and Taku Noda

***Abstract*— Steady-state simulation plays a vital role in power system analysis and design. Steady-state initialization is important for the startup of an electromagnetic transient simulation. One of the commonly used steady-state analysis is phasor analysis. If the system is free from harmonics, phasor analysis is accurate. However, when a nonlinear or time-varying component is present in the system, phasor analysis is inadequate because the impact of the harmonics on the operating point is not taken into account. This paper presents a new power flow algorithm based on both time and frequency domains, which is also called the hybrid analysis. The proposed power flow algorithm employs a time-domain method to model nonlinear components and time-varying components and uses a frequency domain method to handle linear and distributed elements. Although in the past work, hybrid approach has been used to obtain steady-state voltage and current waveforms of a nonlinear or time-varying system, it has not yet been used for a power flow analysis. In this paper, we will show how to extend the existing hybrid method to account for the power flow constraints, imposed by the generator and load buses. Moreover, the proposed method is more efficient, as compared to the existing method because one redundant Newton iteration loop is eliminated in the proposed hybrid method.**

***Index Terms*—Time-domain analysis, frequency-domain analysis, Newton's method, Power system harmonics**

I. INTRODUCTION

STEADY-state analysis plays a vital role in power system design and analysis: A steady-state operating point is necessary for small-signal analysis, which is commonly used for controller design and stability evaluation. Moreover, accurate steady-state initializations are important for the startup of an electromagnetic transient simulation.

When systems only contain linear time invariant (LTI) components, phasor analysis can provide accurate steady-state solutions. However, when a nonlinear or time-varying component exists in a system, phasor analysis becomes inadequate. In general, there are two approaches for obtaining the steady-state solution of a system — time- and frequency-domain approaches. Time-domain steady-state analysis, also called the shooting method, is pioneered by Aprille and Trick [1]. They recognized that finding the periodic steady-state solution of a circuit can be formulated as a two-point boundary-value problem in which the solution over one period T, is required to satisfy the periodicity constraint:

$$x(T) - x(0) = 0\ , \tag{1}$$

where x is a vector, containing the system states. Based on (1), a Newton-type algorithm can be derived, and it can rapidly find out the steady-state solution [1]. Newton's method requires the evaluation of a Jacobian matrix. The method of sensitivity circuit analysis [2] allows the Jacobian matrix to be constructed by simulating, over one period, a set of companion circuits, via transient analysis. Such a feature allows the shooting method to be easily incorporated into a transient simulation program [3]. However, the shooting method is not without its disadvantages. The shooting method cannot handle distributed elements [4]. Therefore, transmission lines cannot be represented by distributed-parameter line equivalents and have to be represented by their single or cascaded pi equivalents. If cascaded pi equivalents are used to gain accuracy, the computation efficiency will be rapidly decreased because the number of inductors and capacitors is extensive, requiring a large number of transient simulations for acquiring the Jacobian matrix [5]. Frequency domain methods [6]-[12] use an entirely different approach. Essentially, all the system state variables are represented by their Fourier coefficients so that the differential equations of the system can be transformed into complex algebraic equations. These equations are then solved by an algorithm of Newton-type [7]-[12]. Linear or distributed elements such as transmission lines can all be easily handled in the frequency domain. However, the main problem of the frequency-domain methods is that a large number of harmonics may need to be taken into considerations if the system consists of components with strong nonlinearities [13].

Both time- and frequency-domain approaches have their advantages and disadvantages. Therefore, the best way is to solve the system by a combination of the two approaches. This is commonly referred to as the hybrid methods. Yacamini et al. [14] and Arrillaga et al. [15], [16] were one of the pioneers who used the hybrid methods to obtain the steady-state solution of a power system. The system they studied was a Thyristor-based AC-DC converter. The algorithm is as follows:

The system to be solved is assumed to be at a certain operating

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point and the corresponding time-domain ac current and dc voltage waveforms are derived. Then, these waveforms are converted into the frequency domain, and the dc currents are solved and converted back to the time domain. The iteration process continues until the desired operating point is found. Semlyen and Medina [17], and Ushida et al. [5] extended this hybrid concept and applied it to a general power system. They proposed to divide a power system into two parts such that the "frequency-friendly" components such as linear R, L, C, and distributed transmission lines are modeled in the frequency domain while "time-domain friendly" devices such as nonlinear R, L, C, and time-varying devices are modeled in the time domain. Moreover, instead of using the sequential iteration as in [14]-[16], Newton's iteration is employed to achieve quadratic convergence [18]. Fig. 1 demonstrates the basic ideas of the hybrid method proposed in [17] and [5]. First, the system is partitioned into two parts. Both parts are connected to a common harmonic voltage source, V, as shown in Fig. 1. Note that for illustration purpose, Fig. 1 only shows two-port systems. The method can be extended to the multi-port cases. One part of the system is solved by the time-domain shooting method, while the other in the frequency domain to obtain the injected current harmonics I_1 and I_2, respectively. Then, the Newton iteration can be formulated by forcing the sum of the two injected currents to be zero (i.e. Kirchhoff's current law):

$$\zeta^{(k+1)} = \zeta^{(k)} + \Delta\zeta^{(k)}, \text{ for } k = 1, 2, 3, \cdots \tag{2}$$

where $\zeta = V$.

The correction term, $\Delta\zeta^{(k)}$ is given by the solution of

$$J^{(k)}\Delta\zeta^{(k)} = M^{(k)}, \tag{3}$$

where $M = I_1 + I_2$, and $J = \frac{\partial M}{\partial \zeta}$.

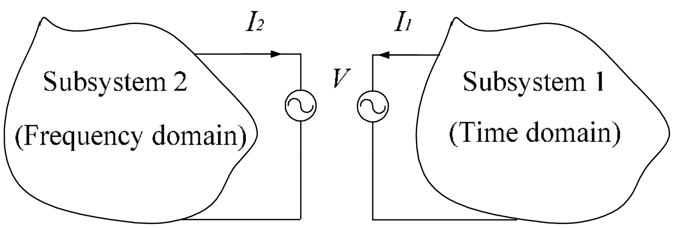

Fig. 1. Algorithm of the existing hybrid method

Such an algorithm is conceptually elegant but may not be very efficient. This is because I_1 in (2) is obtained after the shooting method is applied to the time-domain subsystem. Therefore, a double iteration loop is resulted. If 10 iterations are required by the shooting method, and 10 iterations required by (2) to reach the final convergence, then a total of 100 iterations are required. Such a number may well reach that required by a brute force time-domain transient analysis for obtaining the steady-state solutions. Moreover, the hybrid algorithm presented in [5] and [17] only deals with how to obtain the steady-state waveforms of the power system. In a power system, not only steady-state waveforms but also the values of the active and reactive power components delivered or absorbed by the generator and load buses are interested by the power engineers. Note that [16] has added the power flow constraints to the formulation proposed in [14] and [15]. However, how power flow constraints can be added to the formulation proposed in [5] and [17] have not yet been discussed. Hence, the main objective of this paper is to modify the hybrid method proposed in [5] and [17] (i.e. equation (2) and (3)) such that it becomes more efficient and is able to account for power flow constraints imposed by the generator and load buses.

The organization of this paper is as follows: Section II describes the proposed hybrid algorithm. Section III shows two numerical examples. Finally, a conclusion is given in Section IV.

II. Proposed Hybrid Algorithm

A. Formulation of the Proposed Method

The reason that the hybrid method proposed in [5] and [17] requires two iteration loops is that the shooting method is used before (2) and (3) are evaluated. However, such a loop is redundant and can be eliminated. To eliminate this loop, we simply extract the periodicity constraints of subsystem 1 in Fig. 1 and put it in the mismatch vector, M of (3). In this way, we are forcing the Kirchhoff's current law and the periodicity constraints to be simultaneously satisfied. Hence, only one loop is resulted. Moreover, to satisfy the power flow constraints imposed by the generator and load buses, the active power and reactive power constraints from both two subsystems are also added to M of (3). Fig. 2 illustrates the concept of the proposed method.

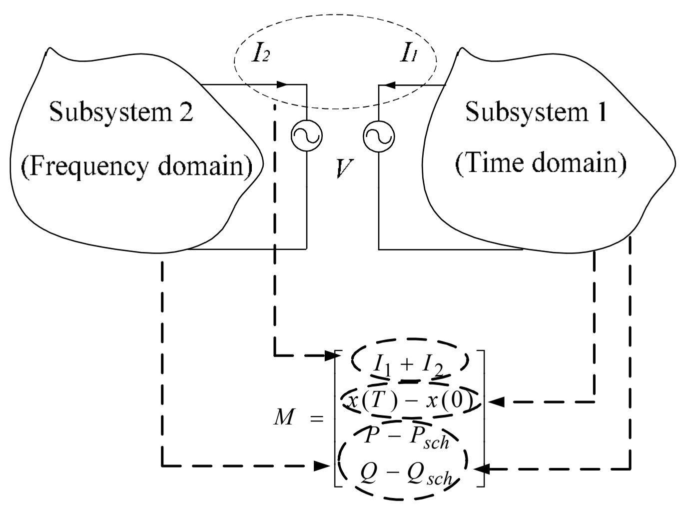

Fig. 2. Algorithm of the proposed hybrid method

Following the concept of Fig. 2, equation (4) is resulted.

$$\zeta^{(k+1)} = \zeta^{(k)} + \Delta\zeta^{(k)}, \text{ for } k = 1, 2, 3, \cdots \tag{4}$$

where $\zeta = \begin{bmatrix} \mathrm{Re}\{V\} & \mathrm{Im}\{V\} & x(0) & e_{abs} & \theta \end{bmatrix}^T$.

The correction term, $\Delta\zeta^{(k)}$ is given by the solution of

$$J^{(k)}\Delta\zeta^{(k)} = M^{(k)}, \tag{5}$$

where

$M = [\mathrm{Re}\{I_1 + I_2\} \quad \mathrm{Im}\{I_1 + I_2\} \quad x(T) - x(0) \quad P - P_{sch} \quad Q - Q_{sch}]^T$, $J = \dfrac{\partial M}{\partial \zeta}$, e_{abs} is a vector, containing the voltage magnitudes of the PQ buses, θ the phase angles of the PV and PQ buses, P_{sch} the scheduled active power components of the PV and PQ buses, and Q_{sch} the scheduled reactive power components of the PQ buses. Note that the mismatch equations presented in Fig. 2 are a mixture of real and complex numbers. Thus, all the complex quantities (the voltage and current harmonics) need to be decomposed into real and imaginary components as shown in (4) and (5).

B. Calculation of the Jacobian Matrix

The Jacobian matrix of (5) is expressed in (6). Some of the elements in (6) can be obtained analytically while some of them are easier to be evaluated numerically. For instance, $\dfrac{\partial \mathrm{Re}\{I_2\}}{\partial \mathrm{Re}\{V\}}$, $\dfrac{\partial \mathrm{Re}\{I_2\}}{\partial \mathrm{Im}\{V\}}$, $\dfrac{\partial \mathrm{Im}\{I_2\}}{\partial \mathrm{Re}\{V\}}$, and $\dfrac{\partial \mathrm{Im}\{I_2\}}{\partial \mathrm{Re}\{V\}}$ can be directly obtained by inspecting the admittance matrix of the frequency domain subsystem [19]. On the other hand, all the other terms are easier to be evaluated numerically: $\dfrac{\partial x(T)}{\partial x(0)}$ can be obtained by simulating, over one period, a set of companion circuits, via transient analysis [2]. The rest of the terms can be obtained by sequentially perturbing each element of the unknown variables, and calculating the change in the corresponding variables or mismatch equations.

The numerical calculation of the Jacobian has the advantage of ease of coding, but it is slow. Since the majority of the Jacobian entries are calculated numerically, it is not computationally efficient to re-evaluate the Jacobian matrix for each iteration. Thus, we propose to use Broyden's method [20] to update the Jacobian matrix. In essence, the full Jacobian matrix is only calculated once to start the iteration process. Refinement of the Jacobian matrix is achieved by introducing correction terms in the iteration loop. The disadvantage of Broyden's method is that the quadratic convergence of Newton's method is lost, being replaced by the superlinear convergence [20]. However, the reduction to superlinear convergence is justified due to the greater reduction in the amount of the computation time at each iteration step.

C. Power Electronic Modeling

The concept that linear and distributed elements are modeled in the frequency domain while nonlinear and time-varying devices modeled in the time domain, is generally true. However, one needs to pay particular attention to one type of time-varying devices — power electronic devices. In order for a power electronic device to work properly, there is an important requirement (Kirchhoff's laws) needs to be followed: The dc side of a voltage source converter (VSC) needs to appear as a voltage source and the ac side needs to appear as current source, and vice versa for a current source converter (CSC) [21]. Thus, the capacitor at the dc side and the inductors at the ac side of a VSC whereas the inductor at dc side and the capacitors at the ac side of a CSC cannot be absorbed into the frequency domain. They should be regarded as an integral part of these devices, and modeled in the time domain.

III. NUMERICAL EXAMPLES

To validate the proposed algorithm, two numerical examples are presented in this section. To simplify the problem, all the generator and load buses considered are represented by ideal voltage sources.

A. Example 1

Fig. 3 shows a three-bus system. Bus 1 is designated to be the slack bus, Bus 2 is connected to a VSC, whose switching frequency is 1620Hz, and Bus 3 is connected to a PQ load. Transmission lines, TL_{12}, TL_{13}, and TL_{23} are assumed to be the same length. All the parameter values are listed in the Appendix section.

$$J = \frac{\partial M}{\partial \zeta} = \begin{bmatrix} \frac{\partial \mathrm{Re}\{I_1+I_2\}}{\partial \mathrm{Re}\{V\}} & \frac{\partial \mathrm{Re}\{I_1+I_2\}}{\partial \mathrm{Im}\{V\}} & \frac{\partial \mathrm{Re}\{I_1\}}{\partial x(0)} & \frac{\partial \mathrm{Re}\{I_1+I_2\}}{\partial e_{abs}} & \frac{\partial \mathrm{Re}\{I_1+I_2\}}{\partial \theta} \\ \frac{\partial \mathrm{Im}\{I_1+I_2\}}{\partial \mathrm{Re}\{V\}} & \frac{\partial \mathrm{Im}\{I_1+I_2\}}{\partial \mathrm{Im}\{V\}} & \frac{\partial \mathrm{Im}\{I_1\}}{\partial x(0)} & \frac{\partial \mathrm{Im}\{I_1+I_2\}}{\partial e_{abs}} & \frac{\partial \mathrm{Im}\{I_1+I_2\}}{\partial \theta} \\ \frac{\partial \{x(T)-x(0)\}}{\partial \mathrm{Re}\{V\}} & \frac{\partial \{x(T)-x(0)\}}{\partial \mathrm{Im}\{V\}} & \frac{\partial x(T)}{\partial x(0)} - I & \frac{\partial \{x(T)-x(0)\}}{\partial e_{abs}} & \frac{\partial \{x(T)-x(0)\}}{\partial \theta} \\ \frac{\partial P}{\partial \mathrm{Re}\{V\}} & \frac{\partial P}{\partial \mathrm{Im}\{V\}} & \frac{\partial P}{\partial x(0)} & \frac{\partial P}{\partial e_{abs}} & \frac{\partial P}{\partial \theta} \\ \frac{\partial Q}{\partial \mathrm{Re}\{V\}} & \frac{\partial Q}{\partial \mathrm{Im}\{V\}} & \frac{\partial Q}{\partial x(0)} & \frac{\partial Q}{\partial e_{abs}} & \frac{\partial Q}{\partial \theta} \end{bmatrix} \tag{6}$$